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Some Common Fixed Point Theorems in 2-Metric Spaces

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ABSTRACT

In this paper, we obtain some results of fixed point theorems in 2-metric spaces which are inspired by the works of V. Gupta *et al.*[3]. The results are proved using some binary relation and conditions on the mappings. Existence and uniqueness of fixed points of self maps satisfying certain conditions are investigated in a complete 2-metric space.

Keywords: Fixed point, 2-metric space, weak compatibility etc.

Let X be a non-empty set and let d: $X \times X \times X \to [0,\infty)$ be such that,

- ❖ To each pair of point x, y in X with $x \neq y$ there exists a point z in X such that $d(x,y,z) \neq 0$.
- ❖ $d(x,y,z) = 0$ when at least two of the three points are equal.
- ❖ For any x,y,z in X , $d(x,y,z) = d(x,z,y) = d(y,z,x)$.
- ❖ For any x,y,z,w in X , $d(x,y,z) \leq d(x,y,w) + d(x,w,z) + d(w,y,z)$,
- ❖ Then d is called a 2-metric[2] and (X,d) is called a 2-metric space[2].

In this note X will denote a complete 2-metric space unless or otherwise stated instead of (X,d).

- ❖ A sequence $\{x_n\}$ in X is called a Cauchy sequence[7] when $d(x_n,x_m,a) \to 0$ as $n,m \to \infty$
- ❖ A sequence $\{x_n\}$ in X is said to be converge[7] to an element x in X when $d(x_n,x,a) \to 0$ as $n \to \infty$

It is interesting to note that every convergent sequence in a 2-metric space need not be a Cauchy sequence[7]. A 2-metric d is said to be continuous when it is continuous in two of its arguments[7]. The notion of weak commutivity compatibility, weakly compatibility analogous introduced in 2-metric spaces {[1],[9]} as they are available in metric space{[4],[5],[6]}.

The notion of binary relation has been used in[3] and some common fixed point theorems have been obtained in 2-metric spaces.

In this paper we have made attempt to obtain some common fixed point theorems for four mappings in 2-metric spaces. Before going to state and prove the main theorem we collect the following definitions[3]:

- **Definition 1**: Let A and B be mappings from a metric space (X,d) into itself. A and B are said to be weakly compatible if they commute at their coincidence point i.e., Ax = Bx for some x in X implies ABx = BAx.
- **Definition 2:** Let $\Diamond: R^+ \times R^+ \rightarrow R^+$ be a binary operation satisfying the following conditions:
  1. $\Diamond$ is associative and commutative
  2. $\Diamond$ is continuous.
- **Definition 3:** the binary operation $\Diamond$ is said to satisfy α-property if there exists a positive real number α such that $a \Diamond b \leq \alpha\{ a,b\}$ for all $a,b \in R^+$.

Main result

Theorem: Let (X,d) be a complete 2-metric space such that $\Diamond$ satisfy α-property with $\alpha \geq 0$. Let A, B, S, T be self-mappings of X into itself satisfy following conditions:

a. A(X) T(X), B(X) S(X) and S(X), T(X) are closed sub sets of X.
b. The pair (A,S) and (B,T) are weakly compatible.
c. $dAx,By,u) \leq K_1 d(Sx,Ty,u)\Diamond d(Ax,Sx,u)] + K_2[d(Sx,Ty,u)\Diamond d(By,Ty,u)] + K_3[d(Sx,Ty,u) \Diamond d(Ax,By,u)] + K_4[d(Sx,Ty,u) \Diamond d(Ax,Ty,u] + K_5[d(Sx,Ty,u) \Diamond \{d(Ax,By,u) + d(By,Ty,u)\}] + K_6[d(Sx,Ty,u) \Diamond \{d(Ax,Sx,u) + d(By,Ty,u)\}]$

for all x, y in X, where $K_1, K_2, K_3, K_4, K_5, K_6 \geq 0$ and $\sum_{i=1}^{6} K_i < 1$. Then A, B, S, T have a unique common fixed point in X.

Proof: Let x_0 be an arbitrary point in X. We can find deductively a sequence $\{y_n\}$ in X such that $y_{2n} = Ax_{2n} = Tx_{2n+1}$ and $y_{2n+1} = Bx_{2n+1} = Sx_{2n+2}$, for n = 0, 1, 2, 3,…

We claim that $\{y_n\}$ is a Cauchy sequence using (c) we get,

$$d(y_{2n},y_{2n+1},u) = d(Ax_{2n},Bx_{2n+1},u) \leq K_1[d(Sx_{2n},Tx_{2n+1},u) \Diamond d(Ax_{2n},Sx_{2n},u) + K_2[d(Sx_{2n},Tx_{2n+1},u) \Diamond d(Bx_{2n+1},Tx_{2n+1},u)] + K_3[[d(Sx_{2n},Tx_{2n+1},u)) \Diamond d(Ax_{2n},Bx_{2n+1},u)] + K_4[d(Sx_{2n},Tx_{2n+1},u) \Diamond d(Ax_{2n},Tx_{2n+1},u)] + K_5[d(Sx_{2n},Tx_{2n+1},u)) \Diamond \{d(Ax_{2n},Bx_{2n+1},u) + d(Bx_{2n+1},Tx_{2n+1},u)\}] + K_6[[d(Sx_{2n},Tx_{2n+1},u) \Diamond \{d(Ax_{2n},Sx_{2n},u) + d(Bx_{2n+1},Tx_{2n+1},u)\}]$$

$$= K_1[d(y_{2n-1},y_{2n},u) \Diamond d(y_{2n},y_{2n-1},u)] + K_2[d(y_{2n-1},y_{2n},u) \Diamond d(y_{2n+1},y_{2n}, u)] + K_3[d(y_{2n-1},y_{2n},u) \Diamond d(y_{2n},y_{2n+1},u)] + K_4[d(y_{2n-1},y_{2n},u) \Diamond d(y_{2n},y_{2n,}u] + K_5[d(y_{2n-1},y_{2n},u) \Diamond \{d(y_{2n},y_{2n+1},u + d(y_{2n+1},y_{2n},u)\}] + K_6[d(y_{2n-1},y_{2n},u) \Diamond\{d(y_{2n},y_{2n-1},u) + d(y_{2n+1},y_{2n}, u)\}].$$

Let $d_n = d(y_{n-1},y_n,u)$. Then from above inequality we get,

$$d_{2n+1} \leq K_1[d_{2n} \Diamond d_{2n}] + K_2[d_{2n} \Diamond d_{2n+1}] + K_3[d_{2n} \Diamond d_{2n+1}] + K_4[d_{2n} \Diamond 0] + K_5[d_{2n} \Diamond ½\{d_{2n+1} + d_{2n+1}\}] + K_6[d_{2n} \Diamond ½\{d_{2n} + d_{2n+1}\}]$$

i.e., $d_{2n+1} \leq \alpha K_1 d_{2n} + \alpha K_2 \max\{d_{2n}, d_{2n+1}\} + \alpha K_3 \max\{d_{2n}, d_{2n+1}\} + \alpha K_4 d_{2n} + \alpha K_5 \max\{d_{2n}, d_{2n+1}\} + \alpha K_6 \max\{d_{2n}, (d_{2n} + d_{2n+1})\}$ …(1)

Let, if possible that, $d_{2n+1} > d_{2n}$.

Then from (1) we get,

$$d_{2n+1} \leq \alpha K_1 d_{2n} + \alpha K_2 d_{2n+1} + \alpha K_3 d_{2n+1} + \alpha K_4 d_{2n} + \alpha K_5 d_{2n+1} + \alpha K_6 d_{2n+1}.$$

Or, $d_{2n+1} < \alpha (K_1 + K_2 + K_3 + K_4 + K_5 + K_6) d_{2n+1} < d_{2n+1}$, [as $\alpha(K_1+K_2+K_3+K_4+K_5+K_6) <1$],

Which is a contradiction.

So, $d_{2n+1} < d_{2n}$ i.e., $d_{2n} < d_{2n-1}$;

Therefore, $d_{2n} < d_{n-1}$, for n = 1,2,3….

So, $d_n < \alpha(K_1 + K_2 + K_3 + K_4 + K_5 + K_6) d_{n-1}$ i.e., $d_n < K d_{n-1}$ where,

$$K = \alpha(K_1 + K_2 + K_3 + K_4 + K_5 + K_6) <1$$

By iteration n times we get,

$$d_n < K d_{n-1} < K^2 d_{n-2} < \ldots < K^n d_0$$

Taking lim as $n \to \infty$ we get, $\lim_{n\to\infty} d_n = 0$

So, $\lim_{n\to\infty} d(y_{n-1},y_n,u) = 0$ …(2)

Let, m>n where m = 2n+1.

We prove $\{y_n\}$ is a Cauchy sequence by the method of contradiction.

Let, if possible suppose that n is the least integer for which $d(y_n,y_m,u) \geq \varepsilon$ but, $d(y_{n-1},y_m,u) < \varepsilon$

Now, $\varepsilon < d(y_n,y_m,u) \leq d(y_n,y_m,y_{n-1}) + d(y_n,y_{n-1},u) + d(y_{n-1},y_m,u)$ …(3)

Now, $d(y_n,y_m,y_{n-1}) = d(Ax_n,Bx_m,y_{n-1}) \leq K_1[d(Sx_n,Tx_m,y_{n-1}) \Diamond d(Ax_n,Sx_n,y_{n-1})] + K_2[d(Sx_n,Tx_m,y_{n-1}) \Diamond d(Bx_m,Tx_m,y_{n-1})] + K_3[d(Sx_n,Tx_m,y_{n-1}) \Diamond d(Ax_n,Bx_m,y_{n-1})] + K_4[d(Sx_n,Tx_m,y_{n-1}) \Diamond d(Ax_n,Tx_m,y_{n-1}] + K_5[d(Sx_n,Tx_m,y_{n-1}) \Diamond ½\{d(Ax_n,Bx_m,y_{n-1}) + d(Bx_m,Tx_m,y_{n-1})\}] + K_6[d(Sx_n,Tx_m,y_{n-1}) + ½\{d(Ax_n,Sx_n,y_{n-1}) + d(Bx_m,Tx_m,y_{n-1})\}]$

$= K_1[d(y_{n-1},y_{m-1},y_{n-1})◊d(y_n,y_{n-1},y_{n-1})] + K_2[d(y_{n-1},y_{m-1},y_{n-1})◊d(y_m,y_{m-1},y_{n-1})] + K_3[d(y_{n-1},y_{m-1},y_{n-1})◊d(y_n,y_m,y_{n-1})] + K_4[d(y_{n-1},y_{m-1},y_{n-1})◊d(y_n,y_{m-1},y_{n-1}] + K_5[d(y_{n-1},y_{m-1},y_{n-1})◊½\{d(y_n,y_m,y_{n-1}) + d(y_m,y_{m-1},y_{n-1})\}] + K_6[d(y_{n-1},y_{m-1},y_{n-1}) + ½\{d(y_n,y_{n-1},y_{n-1}) + d(y_m,y_{m-1},y_{n-1})\}]$

Or, $d(y_n,y_m,y_{n-1}) \le K_2\, d(y_m,y_{m-1},y_{n-1}) + K_3\, d(y_n,y_m,y_{n-1}) + K_4\, d(y_n,y_{m-1},y_{n-1}) + K_5 ½\{d(y_n,y_m,y_{n-1}) + d(y_m, y_{m-1}, y_{n-1})\}] + K_6 ½ d(y_m,y_{m-1},y_{n-1})$

Using (2) and taking lim $n \to \infty$ we get, $d(y_n,y_m,y_{n-1}) = 0$ …(4)

Using (2) and (4), we get from (3)

$\varepsilon < 0 + 0 + d(y_{n-1},y_m,u) < \varepsilon$ i.e., $\varepsilon < \varepsilon$

Which is a contradiction.

Hence, $\{y_n\}$ is a Cauchy sequence.

Since, X is a complete 2-metric space.

Therefore, lim $n \to \infty$ $y_n = y$ in X.

Hence, $\lim_{n \to \infty} y_n = \lim_{n \to \infty} Ax_{2n} = \lim_{n \to \infty} Bx_{2n+1}$

$= \lim_{n \to \infty} Sx_{2n+2} = \lim_{n \to \infty} Tx_{2n+1} = y$ …(5)

Now, since *T(X)* is a closed subset of *X*, there exists a *v* in *X* such that, $Tv = y$ …(6)

If $Bv \neq y$ then by using (c) we get,

$d(Ax_{2n},Bv,u) \le K_1[d(Sx_{2n},Tv,u)◊d(Ax_{2n},Sx_{2n},u)] + K_2[d(Sx_{2n},Tv,u)◊d(Bv,Tv,u)] + K_3[d(Sx_{2n},Tv,u)◊d(Ax_{2n},Bv,u)] + K_4[d(Sx_{2n},Tv,u)◊d(Ax_{2n},Tv,u] + K_5[d(Sx_{2n},Tv,u)◊½\{d(Ax_{2n},Bv,u) + d(Bv,Tv,u)\}] + K_6[d(Sx_{2n},Tv,u)◊½\{d(Ax_{2n},Sx_{2n},u) + d(Bv,Tv,u)\}]$

Taking lim as $n\to\infty$ on both side we get,

$d(y,Bv,u) \le K_1[d(y,y,u)◊d(y,y,u)] + K_2[d(y,y,u)◊d(Bv,y,u)] + K_3[d(y,y,u)◊d(y,Bv,u)] + K_4[d(y,y,u)◊d(y,y,u] + K_5[d(y,y,u)◊½\{d(y,Bv,u) + d(Bv,y,u)\}] + K_6[d(y,y,u)◊½\{d(y,y,u) + d(Bv,y,u)\}]$

Or, $d(y,Bv,u) \le \alpha\, K_2\, d(Bv,y,u)] + \alpha\, K_3\, d(y,Bv,u)] + \alpha K_5\, d(y,Bv,u) + \alpha\, K_6\, d(Bv,y,u)$

Or, $d(y,Bv,u) \le \alpha(K_2+K_3+K_5+K_6)\, d(y,Bv,u) < d(y,Bv,u)$ [as $\alpha(K_2+K_3+K_5+K_6)<1$]

Which is a contradiction.

So, Bv = y = Tv

Since, B,T are weakly compatible, we have BTv = TBv i.e., By = Ty. …(7)

Now, if $y \neq By$ then by using (c) we get,

$d(Ax_{2n},By,u) \leq K_1[d(Sx_{2n},Ty,u)\Diamond d(Ax_{2n},Sx_{2n},u)] + K_2[d(Sx_{2n},Ty,u)\Diamond d(By,Ty,u)] + K_3[d(Sx_{2n},Ty,u)\Diamond d(Ax_{2n},By,u)] + K_4[d(Sx_{2n},Ty,u)\Diamond d(Ax_{2n},Ty,u] + K_5[d(Sx_{2n},Ty,u)\Diamond ½\{d(Ax_{2n},By,u) + d(By,Ty,u)\}] + K_6[d(Sx_{2n},Ty,u)\Diamond ½\{d(Ax_{2n},Sx_{2n},u) + d(By,Ty,u)\}]$

Taking lim as $n\to\infty$ on both sides and using (7) and (5) we get,

$d(y,By,u) \leq K_1[d(y,By,u)\Diamond d(y,y,u)] + K_2[d(y,By,u)\Diamond d(By,By,u)] + K_3[d(y,By,u)\Diamond d(y,By,u)] + K_4[d(y,By,u)\Diamond d(y,By,u] + K_5[d(y,By,u)\Diamond ½\{d(y,By,u) + d(By,By,u)\}] + K_6[d(y,By,u)\Diamond ½\{d(y,y,u) + d(By,By,u)\}]$

Or, $d(y,By,u) \leq \alpha(K_1+ K_2+ K_3+ K_4+ K_5+ K_6)\, d(y,By,u) < d(y,By,u)$

[as $\alpha(K_1+ K_2+ K_3+ K_4+ K_5+ K_6) <1$]

Which is a contradiction. Hence, y = By

So, y = By = Ty …(8)

Since, $B(X)\subseteq S(X)$ there exists w in X, such that Sw = y.[As By = y] …(9)

Now, if $Aw \neq y$ then using (C),

$d(Aw,By,u) \leq K_1[d(Sw,Ty,u)\Diamond d(Aw,Sw,u)] + K_2[d(Sw,Ty,u)\Diamond d(By,Ty,u)] + K_3[d(Sw,Ty,u)\Diamond d(Aw,By,u)] + K_4[d(Sw,Ty,u)\Diamond d(Aw,Ty,u] + K_5[d(Sw,Ty,u)\Diamond ½\{d(Aw,By,u) + d(By,Ty,u)\}] + K_6[d(Sw,Ty,u)\Diamond ½\{d(Aw,Sw,u) + d(By,Ty,u)\}]$.

Using (8) and (9) we get,

$d(Aw,y,u) \leq K_1[d(y,y,u)\Diamond d(Aw,y,u)] + K_2[d(y,y,u)\Diamond d(y,y,u)] + K_3[d(y,y,u)\Diamond d(Aw,y,u)] + K_4[d(y,y,u)\Diamond d(Aw,y,u] + K_5[d(y,y,u)\Diamond ½\{d(Aw,y,u) + d(y,y,u)\}]+ K_6[d(y,y,u)\Diamond ½\{d(Aw,y,u) + d(y,y,u)\}]$.

Or, $d(Aw,y,u) \leq \alpha(K_1+K_3+K_4+K_5/2+K_6/2)\, d(Aw,y,u) < \alpha(K_1+K_3+K_4+K_5+K_6)\, d(Aw,y,u) < d(Aw,y,u)$,

Which is a contradiction.

Hence, Aw = y implies Sw = y = Aw.

Since, S and A are weakly compatible, ASw = SAw implies Sy = Ay. …(10)

Now, if $Ay \neq y$ then by using (C) we get,

$d(Ay,y,u) = d(Ay,By,u) \leq K_1[d(Sy,Ty,u)\Diamond d(Ay,Sy,u)] + K_2[d(Sy,Ty,u)\Diamond d(By,Ty,u)] + K_3[d(Sy,Ty,u)\Diamond d(Ay,By,u)] + K_4[d(Sy,Ty,u)\Diamond d(Ay,Ty,u)] + K_5[d(Sy,Ty,u)\Diamond ½\{d(Ay,By,u) + d(By,Ty,u)\}] + K_6[d(Sy,Ty,u)\Diamond ½ d(Ay,Sy,u) + d(By,Ty,u)\}]$.

Using (8) and (10) we get, $d(Ay,y,u) \leq K_1[d(Ay,y,u)\Diamond d(Ay,Ay,u)] + K_2[d(Ay,y,u)\Diamond d(y,y,u)] + K_3[d(Ay,y,u)\Diamond d(Ay,y,u)] + K_4[d(Ay,y,u)\Diamond d(Ay,y,u)] + K_5[d(Ay,y,u)\Diamond ½\{d(Ay,y,u) + d(y,y,u)\}] + K_6[d(Ay,y,u)\Diamond ½\{d(Ay,Ay,u) + d(y,y,u)\}]$.

Or, $d(Ay,y,u) \leq \alpha(K_1+ K_2+ K_3+ K_4+ K_5+ K_6)\, d(Ay,y,u)< d(Ay,y,u)$,

Which is a contradiction. So, $Ay = y$.

Using $Ay = y = Sy$ and from (8) we get, $Ay = By = Sy = Ty = y$. …(11)

i.e., y is a common fixed point for A, B, S, T.

we now show that y is a unique common fixed point of A,B,S and T.

Let, x be another fixed point of A,B,S,T and $x \neq y$

Then $d(x,y,u)= d(Ax,By,u) \leq K_1[d(Sx,Ty,u)\Diamond d(Ax,Sx,u)] + K_2[d(Sx,Ty,u)\Diamond d(By,Ty,u)] + K_3[d(Sx,Ty,u)\Diamond d(Ax,By,u)] + K_4[d(Sx,Ty,u)\Diamond d(Ax,Ty,u] + K_5[d(Sx,Ty,u)\Diamond ½\{ d(Ax,By,u) + d(By,Ty,u)\}] + K_6[d(Sx,Ty,u)\Diamond ½\{d(Ax,Sx,u) + d(By,Ty,u)\}]$

i.e., $d(x,y,u) \leq K_1[d(x,y,u)\Diamond d(x,x,u)] + K_2[d(x,y,u)\Diamond d(y,y,u)] + K_3[d(x,y,u)\Diamond d(x,y,u)] + K_4[d(x,y,u)\Diamond d(x,y,u] + K_5[d(x,y,u)\Diamond ½\{d(x,y,u) + d(y,y,u)\}] + K_6[d(x,y,u)\Diamond ½\{d(x,x,u) + d(y,y,u)\}]$

i.e., $d(x,y,u) \leq \alpha(K_1+ K_2+ K_3+ K_4+ K_5+ K_6)\, d(x,y,u)< d(x,y,u)$,

Which is a contradiction.

So, $x = y$.

Hence, A, B, S, T have a unique common fixed point.

We have the following corollaries:

Corollary 1: Let (X,d) be a 2-metric space such that $\Diamond$ satisfy α-property with $\alpha \geq 0$. Let A, B and S be self mappings of X into itself satisfy following conditions:

a. A(X) S(X), B(X) S(X) and S(X) is a closed sub sets of X.
b. The pair (A,S) and (B,S) are weakly compatible.

c. $d(Ax,By,u) \leq K_1[d(Sx,Sy,u)\Diamond d(Ax,Sx,u)] + K_2[d(Sx,Sy,u)\Diamond d(By,Sy,u)] + K_3[d(Sx,Sy,u)\Diamond d(Ax,By,u)] + K_4[d(Sx,Sy,u)\Diamond d(Ax,Sy,u] + K_5[d(Sx,Sy,u)\Diamond ½\{d(Ax,By,u) + d(By,Sy,u)\}] + K_6[d(Sx,Sy,u)\Diamond ½\{d(Ax,Sx,u) + d(By,Sy,u)\}]$

For all x, y in X, where $K_1, K_2, K_3, K_4, K_5, K_6 \geq 0$ and $\sum_{i=1}^{6} K_i < 1$. Then A,B and S have a unique common fixed point in X.

Proof: Put S=T in the main theorem and get the result.

Corollary 2: Let (X,d) be a complete 2-metric space such that ◊ satisfy α-property with $\alpha \geq 0$. Let A and B be self-mappings of X into itself satisfy following conditions:

$d(Ax,By,u) \leq K_1[d(x, y,u)\Diamond d(Ax,x,u)] + K_2[d(x,y,u)\Diamond d(By,y,u)] + K_3[d(x,y,u)\Diamond d(Ax,By,u)] + K_4[d(x,y,u)\Diamond d(Ax,y,u] + K_5[d(x,y,u)\Diamond ½\{d(Ax,By,u) + d(By,y,u)\}] + K_6[d(x,y,u)\Diamond ½\{d(Ax,x,u) + d(By,y,u)\}]$

for all x, y in X, where $K_1, K_2, K_3, K_4, K_5, K_6 \geq 0$ and $\sum_{i=1}^{6} K_i < 1$. Then A and B have a unique common fixed point in X.

Proof: Put S = I in corollary 1 and get the result.

Corollary 3: Let (X,d) be a complete 2-metric space such that ◊ satisfy α-property with $\alpha \geq 0$. Let A be self mappings of X into itself satisfy following conditions:

$d(Ax,Ay,u) \leq K_1[d(x, y,u)\Diamond d(Ax,x,u)] + K_2[d(x,y,u)\Diamond d(Ay,y,u)] + K_3[d(x,y,u)\Diamond d(Ax,Ay,u)] + K_4[d(x,y,u)\Diamond d(Ax,y,u] + K_5[d(x,y,u)\Diamond ½\{d(Ax,Ay,u) + d(Ay,y,u)\}] + K_6[d(x,y,u)\Diamond ½\{d(Ax,x,u) + d(Ay,y,u)\}]$

for all x, y in X, where $K_1, K_2, K_3, K_4, K_5, K_6 \geq 0$ and $\sum_{i=1}^{6} K_i < 1$. Then, A have a unique common fixed point in X.

Proof: Put B = A in corollary 2 and get the result.

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